Inflation from IIB Superstrings with Fluxes

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We study the conditions needed to have an early epoch of inflationary expansion with a potential coming from IIB superstring theory with fluxes involving two moduli fields. The phenomenology of this potential is different from the usual hybrid inflation scenario and we analyze the possibility that the system of field equations undergo a period of inflation in three different regimes with the dynamics modified by a Randall-Sundrum II term in the Friedmann equation. We find that the system can produce inflation and due to the modification of the dynamics, a period of accelerated contraction can follow or preceed this inflationary stage depending on the sign of one of the parameters of the potential. We discuss on the viability of this model in a cosmological context.

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I. INTRODUCTION

The last 50 years have been one of the most fruitful ones in the life of physics, the standard model of particles and the standard model of cosmology (SMC) essentially developed in this period, are now able to explain a great number of observations in laboratories and cosmological observatories as never before. In only 50 years great steps have been given in the understanding of the origin and development of the universe. Nevertheless, many questions are still open, for example the SMC contains two periods, inflation and structure formation. For the understanding of the structure formation epoch we need to postulate the existence of two kind of substances, the dark matter and the dark energy. Without them, it is impossible to explain the formation of galaxies and cluster of galaxies, or the observed accelerated expansion of the universe. On the other side, it has been postulated a period of inflation in order to give an explanation for several observations as the homogeneity of the universe, the close value of the density of the universe to the critical density or the formation of the seeds which formed the galaxies. However, there is not a theory that unify this two periods, essentially they are disconnected from each other.

In this work we study the possibility that superstring theory could account for the unification between inflation and the structure formation using a specific example. Recently, Frey and Mazumdar [1], were able to compactify the IIB superstrings including 32 fluxes. In the context of the type IIB supergravity theory on the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold with a self-dual three-form fluxes, it has been shown that after compactifaying the effective dilaton-axion potential is given by [1]

$$\begin{aligned} V_{dil} &= \frac{M_P^4}{4(8\pi)^3} h^2\, e^{-2\Sigma_i\sigma_i} \left[e^{-\Phi^{(0)}} \cosh\left(\Phi - \Phi^{(0)}\right) \right. \\ &+ \left. \frac{1}{2}\, e^{\Phi}(C - C^{(0)})^2 - 1 \right], \end{aligned} \tag{1}$$

where $h^2 = \frac{1}{6} h_{mnp} h_{qrs} \delta^{mq}\delta^{nr}\delta^{ps}$. Here h_{mnp} are the NS-NS integral fluxes, the superscript (0) in the fields stands for the fields in the vacuum configuration and finally σ_i with $i = 1, 2, 3$ are the overall size of each factor $\mathbf{T}^2$ of the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold. This potential contains two main scalar fields (moduli fields), the dilaton Φ and the axion C. In [2], the dilaton was interpreted as dark matter and under certain conditions the model reproduces the observed universe, *i.e.*, the structure formation. In this work we investigate if the same theory could give an inflationary period in order to obtain a unified picture between these two epochs. In other words, in this paper we search if it is possible that the same low energy Lagrangian of IIB superstrings with the scalar and axion potential (1) can give an acceptable inflationary period. To deal with the fluxes, we work in the brane representation of space-time, working with a RS-II modification in the equations. Due to the presence of the fluxes during this period, these models have the phenomenology of the Randall-Sundum models [7],[8],[1] and we have chosen to work with the RS-II one, but the same type of analysis could be done for the other model. The paper is organized as follows. In section II we introduce an appropriate parametrization of potential (1) in order to give unities to the physical quantities and to study the cosmology of the low energy Lagrangian and give the field equation to be solved. In section III we solve the equations in the different regimes and conditions. In section IV we give the main results and in section V we discuss some conclusions and perspectives.

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II. THE POTENTIAL

In order to study the cosmology of this model, it is convenient to define the following quantities

$$
\begin{aligned}
\lambda\sqrt{\kappa}\phi &= \Phi - \Phi^{(0)}, \\
V_0 &= \frac{M_P^4}{4(8\pi)^3} h^2\, e^{-2\Sigma_i \sigma_i}\, e^{-\Phi^{(0)}}, \\
C - C^{(0)} &= \sqrt{\kappa}\psi, \\
\psi_0 &= e^{\Phi^{(0)}}, \\
L &= V_0\,(1 - e^{\Phi^{(0)}}), \qquad (2)
\end{aligned}
$$

where λ is the string coupling $\lambda = e^{\langle\Phi\rangle}$, and $\sqrt{\kappa} = 1/M_p$ with $M_p = (8\pi G)^{-1/2}$ the reduced Planck mass. With this new variables, the dilaton potential transforms into

$$
\begin{aligned}
V_{dil} &= V_0\,\left(\cosh\left(\lambda\sqrt{\kappa}\phi\right) - 1\right) + \frac{1}{2} V_0\, e^{\lambda\sqrt{\kappa}\phi} {\psi_0}^2 \kappa \psi^2 + L \\
&= V + L. \qquad (3)
\end{aligned}
$$

where L is interpreted as the cosmological constant, which we will take as subdominant during the inflationary period. In what follows we want to study the behaviour of this potential at early times when the scalar field ϕ takes large values and study the conditions that the parameters λ and V_0 need to meet in order to have inflation. Thus expressing the cosh function in terms of exponentials and taking the limit ϕ big, we arrive to the following expression for the potential:

$$
V(\phi,\psi) = \frac{1}{2} V_0 e^{\lambda\sqrt{\kappa}\phi}(1 + \kappa\psi_0^2\psi^2) - V_0, \qquad (4)
$$

where we have taken into account that the cosmological constant is much smaller than the coefficient of the potential, $L << V_0$, nevertheless, it remains a term V_0 acting during the inflationary period like an extra "cosmological constant" and that we will address as a free parameter of the model. In contrast with the usual hybrid inflation scenario [3], there is no critical value for which this potential exhibits a phase transition triggering the end of inflation (if any such process occurs). The potential follows an exponential behaviour in the ϕ field that prevents it from staying at a fixed value from the start, i.e. it cannot relax at $\phi = 0$ or at any other different value (apart from infinity). A plot of the potential illustrates this behaviour, see Fig. 1. We have checked that there are no saddle points for this potential Eq. (4) and also for potentials (1) and (3) [1], but a global minimum at $\psi = 0$. Therefore, we will assume that there is some mechanism by which the ψ field rolls down to its minimum at $\psi = 0$ oscillating around it at the very early stages of evolution and that any processes such as inflation took place afterwards. In this way, any information

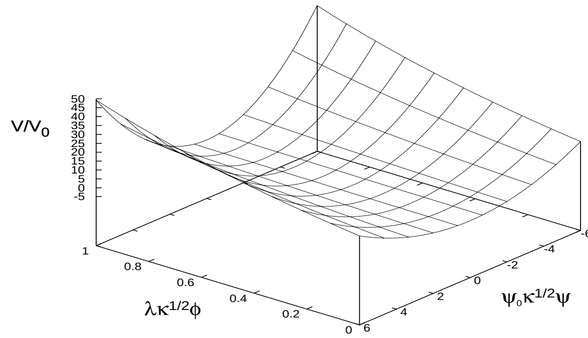

FIG. 1: Potential

concerning the evolution of ψ is erased by the expansion led by the ϕ field if inflation is to happen. Thus we can work with the following expression for the potential:

$$
V(\phi) = \frac{1}{2} V_0 e^{\lambda\sqrt{\kappa}\phi} - V_0 \qquad (5)
$$

We will follow closely the analysis done by Copeland *et al.* [4] and Mendes and Liddle [5] in order to obtain the conditions for this potential to undergo inflation in the cases when the scalar field or the V_0 term dominate the dynamics as well as in the intermediate stage. Our calculations are performed in the high-energy regime within the slow-roll approximation since a potential slow-roll formalism has already been provided for this scenario [10].

A. Field equations

In the presence of branes, particularly in the RS-II scenario, the Friedmann equation changes from its usual expression to [8]:

$$
H^2 = \frac{\kappa}{3}\rho\left(1 + \frac{\rho}{\rho_0}\right), \qquad (6)
$$

where $H \equiv \dot{a}/a$, a is the scale factor of the Universe, a dot means derivative with respect to time and ρ_0 is the brane tension. The total density ρ as well as the equations of motion for the fields in the standard cosmology case are deduced in [9]:

$$
\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2 e^{\lambda\sqrt{\kappa}\phi} + V_\phi + V_\psi e^{\lambda\sqrt{\kappa}\phi}. \qquad (7)
$$

In our case,

$$
V_\phi = \frac{1}{2} V_0 e^{\lambda\sqrt{\kappa}\phi} - V_0, \qquad V_\psi = \frac{1}{2} V_0 \kappa \psi_0^2 \psi^2 \qquad (8)
$$

which in the slow-roll approximation can be written as

$$
H^2 \simeq \frac{\kappa}{3}\left(V_\phi + V_\psi e^{\lambda\sqrt{\kappa}\phi}\right)\left[1 + \frac{\left(V_\phi + V_\psi e^{\lambda\sqrt{\kappa}\phi}\right)}{\rho_0}\right] \qquad (9)
$$

[1] Many thanks to Anupam Mazumdar for this suggestion

The equations of motion for both fields are given considering only the presence of both scalar fields with no radiation fluid, they are [9]

$$\begin{aligned}\ddot{\phi}+3H\dot{\phi}+\frac{\partial V_\phi}{\partial\phi} &= \lambda\sqrt{\kappa}e^{\lambda\sqrt{\kappa}\phi}\left(\frac{1}{2}\dot{\psi}^2-V_\psi\right)\\ \ddot{\psi}+3H\dot{\psi}+\frac{\partial V_\psi}{\partial\psi} &= -\lambda\sqrt{\kappa}\dot{\phi}\dot{\psi}.\end{aligned} \tag{10}$$

Which in the slow-roll approximation are

$$\begin{aligned}3H\dot{\phi}+\frac{\partial V_\phi}{\partial\phi} &\simeq -\lambda\sqrt{\kappa}e^{\lambda\sqrt{\kappa}\phi}V_\psi\\ 3H\dot{\psi}+\frac{\partial V_\psi}{\partial\psi} &\simeq 0.\end{aligned} \tag{11}$$

since the right-hand side of the second equation in (10) can be taken as a kinetic term to the square.

There is an important consequence in the fact that we are considering the ψ field in the value corresponding to the minimum of its potential at $\psi=0$. This leads to a system of equations that do not have source terms and couplings of the fields and we end up with the same system as for the usual RS-II modification. Otherwise, in order to do a proper inflationary analysis, we would have to define new expressions for the potential slow-roll parameters considering not only the modification due to the RS-II term, but also to the couplings between the ψ-potential and the ϕ in the Friedmann equation (9) and the source term in the first of equations (11).

This procedure simplifies considerably the equations but avoids having a realistic analysis of the evolution of both fields including source terms and couplings, something that clearly must have an impact on the conditions to have inflation. Although a full analysis is undoubtedly required, we address the problem with these simplifications as a first insight before more general assumptions are considered.

III. INFLATIONARY PHENOMENOLOGY OF THE MODEL

The analysis that follows will be confined to the high-energy limit of this model, which simplifies calculations further. Consenquently, we have the following system of equations within the slow-roll approximation and the high-energy limit:

$$\begin{aligned}H &\simeq \sqrt{\frac{\kappa}{3\rho_0}}V;\\ 3H\dot{\phi}+\frac{\partial V_\phi}{\partial\phi} &\simeq 0\end{aligned} \tag{12}$$

which can be solved analitically. In this work we take the convention $\dot{\phi}<0$, hence the ϕ field is a decreasing function of time. The solutions of the field equations will be given in section III F.

The expressions for the potential slow-roll parameters for the RS-II cosmology having the inflaton field confined to the brane were deduced by Maartens *et al.* [10]. For the high-energy limit they are:

$$\epsilon\simeq\frac{1}{\kappa}\left(\frac{V'}{V}\right)^2\frac{\rho_0}{V};\qquad \eta\simeq\frac{1}{\kappa}\left(\frac{V''}{V}\right)\frac{\rho_0}{V} \tag{13}$$

where primes indicate derivatives with respect to the ϕ field. The slow-roll approximation is satisfied as long as the slow-roll parameters defined previously accomplish the following conditions :

$$\epsilon\ll 1,\qquad |\eta|\ll 1. \tag{14}$$

The number of e-foldings of inflation in terms of the potential for this model is given in our notation by [10]:

$$N\simeq-\frac{\kappa}{\rho_0}\int_{\phi_N}^{\phi_e}\frac{V^2}{V'}d\phi, \tag{15}$$

where ϕ_N represents the value of ϕ N e-foldings of expansion before the end of inflation and ϕ_e is the value of the field at the end of inflation.

For the potential (5) we have

$$\epsilon=\frac{\lambda^2\rho_0}{4V_0}\frac{e^{2\lambda\sqrt{\kappa}\phi}}{\left(\frac{e^{\lambda\sqrt{\kappa}\phi}}{2}-1\right)^3} \tag{16}$$

$$\eta=\frac{\lambda^2\rho_0}{2V_0}\frac{e^{\lambda\sqrt{\kappa}\phi}}{\left(\frac{e^{\lambda\sqrt{\kappa}\phi}}{2}-1\right)^2} \tag{17}$$

$$\begin{aligned}N\simeq\frac{2V_0}{\lambda^2\rho_0}&\left[-\frac{1}{4}\left(e^{\lambda\sqrt{\kappa}\phi_e}-e^{\lambda\sqrt{\kappa}\phi_N}\right)\right.\\ &\left.+\left(e^{-\lambda\sqrt{\kappa}\phi_e}-e^{-\lambda\sqrt{\kappa}\phi_N}\right)+\lambda\sqrt{\kappa}(\phi_e-\phi_N)\right].\end{aligned} \tag{18}$$

The only way by which inflation can be ended in our model is by violation of the slow-roll approximation with ϵ exceeding unity.

The value of ϕ at which ϵ becomes equal to unity is

$$\begin{aligned}\sqrt{\kappa}\phi_e=\frac{1}{\lambda}\ln\left\{\left[\frac{2\lambda^2\rho_0}{3V_0}\right]\right.&\left[\frac{4^{1/3}B^{1/3}}{2\lambda^2\rho_0}\right.\\ &\left.\left.+\frac{(6V_0+\lambda^2\rho_0)4^{2/3}}{2B^{1/3}}+\frac{(3V_0+\lambda^2\rho_0)}{\lambda^2\rho_0}\right]\right\}\end{aligned} \tag{19}$$

with

$$\begin{aligned}B=\lambda^2\rho_0&\left[27V_0^2+18V_0\lambda^2\rho_0+2\lambda^4\rho_0^2\right.\\ &\left.+(3V_0)^{3/2}\sqrt{4\lambda^2\rho_0+27V_0}\right].\end{aligned} \tag{20}$$

We can rearrange the previous expression so that

$$\begin{aligned}\sqrt{\kappa}\phi_e=\frac{1}{\lambda}\ln\left\{\left[\frac{2\lambda^2\rho_0}{3V_0}\right]\right.&\left[1+\frac{3V_0}{\lambda^2\rho_0}+\frac{2^{2/3}B^{1/3}}{2\lambda^2\rho_0}\right.\\ &\left.\left.+\frac{(6V_0+\lambda^2\rho_0)2^{1/3}}{B^{1/3}}\right]\right\}\end{aligned} \tag{21}$$

So we have that if

$$\left|\frac{3V_0}{\lambda^2\rho_0} + \frac{B^{1/3}}{2^{1/3}\lambda^2\rho_0} + \frac{2^{1/3}(6V_0+\lambda^2\rho_0)}{B^{1/3}}\right| \ll 1, \qquad (22)$$

the ϕ field dominates the potential Eq. (5) and we will have an exponential potential, which in the standard cosmology case corresponds to power-law inflation, but not for the RS-II modification.

The bound found before depends on the choice of values for λ and V_0. As we shall see in section III B, λ is fixed once the value of the brane tension is given. So actually Eq. (22) depends only on the choice of V_0. We use the computing packages Mathematica and Maple to find numerically the values of V_0 ensuring the l.h.s. is real. This happens if:

$$\lambda^2\rho_0 < 0, \implies \qquad V_0 \geq \frac{4}{27}\lambda^2\rho_0 \qquad (23)$$

$$\lambda^2\rho_0 > 0, \implies \qquad V_0 \leq -\frac{1}{6}\lambda^2\rho_0 \qquad (24)$$

Although both intervales guarantee a real value in the roots of condition (22), this is not equivalent to have this condition satisfied in order to have a field dominated region as we shall see later. From these expressions, the second case satisfies having a brane tension with no incompatibilities with nucleosynthesis [5],[13], and we will restrict further calculations to this posibility only.

A. Density perturbations

The field responsible for inflation produces perturbations which can be of three types: scalar, vector and tensor. Vector perturbations decay in an expanding universe and tensor perturbations do not lead to gravitational instabilities producing structure formation. The adiabatic scalar or density perturbations can produce these type of instabilities through the vacuum fluctuations of the field driving the inflationary expansion. So they are usually thought to be the seeds of the large scale structures of the universe [14]. One of the quantities that determines the spectrum of the density perturbations is δ_H which gives the density contrast at horizon crossing (if evaluated at that scale). For the RS-II modification and in our notation, this quantity is [5],[11]

$$\delta(k)^2_H \simeq \frac{\kappa^3}{75\pi^2}\frac{V^3}{V'^2}\frac{V^3}{\rho_0^3}. \qquad (25)$$

Evaluated at the scale $k = aH$. The slow-roll approximation guarantees that δ_H is nearly independent of scale when scales of cosmological interest are crossing the horizon, satisfying the new COBE constrain updated to WMAP3 [16] as $\delta_H = 1.9 \times 10^{-5}$ [15]. Here we take 60 e-foldings before the end of inflation to find the scales of cosmological interest.

We use equation (18),provided we know the value of ϕ_e given by equation (19), to find the value ϕ_N corresponding to $N = 60$, that is 60 e-foldings before the end of inflation and evaluate δ_H as

$$\delta_H^2 \simeq \frac{4\kappa^2}{75\pi^2}\frac{V_0^4}{\lambda^2\rho_0^3}e^{-2\lambda\sqrt{\kappa}\phi_{60}}\left(\frac{e^{\lambda\sqrt{\kappa}\phi_{60}}}{2} - 1\right)^6 \qquad (26)$$

The results are given in table II of section IV

B. Field-dominated region

Considering the case when the ψ field plays no role and ϕ governs the dynamics of the expansion alone, from Eq. (5), we have a potential of exponential type resembling that of power-law inflation in the standard cosmology[12]. The first term of Eq. (5) dominates giving an exponential expansion but not to power-law because the dynamics of RS-II changes this condition. The slow-roll parameters are given by:

$$\epsilon = \eta \simeq \frac{2\rho_0}{V_0}\frac{\lambda^2}{e^{\lambda\sqrt{\kappa}\phi}}. \qquad (27)$$

In contrast with the standard cosmology where they are not only the same but constant. The fact that the dynamics is modified due to the Randall-Sundrum cosmology, allows the existence of a value of ϕ that finishes inflation, since as we just saw, the parameters show a dependence on ϕ and therefore, an evolution.

We have that in this regime, the value of ϕ_e corresponding to the end of inflation is given by $\epsilon \simeq 1$. The $\simeq$ is used because we are in the potential slow-roll approximation not the Hubble one [6].

$$\sqrt{\kappa}\phi_e \simeq \frac{1}{\lambda}\ln\left(\frac{2\lambda^2\rho_0}{V_0}\right) \qquad (28)$$

Inserting this value in the expression for the number of e-foldings (15) for this regime and evaluating, we obtain that ϕ_N with $N = 60$ is

$$\sqrt{\kappa}\phi_{60} \simeq \frac{1}{\lambda}\ln\left(122\frac{\lambda^2\rho_0}{V_0}\right), \qquad (29)$$

and we can evaluate the density contrast Eq (25):

$$\delta_H^2 \simeq \frac{61^4\kappa^2\lambda^6\rho_0}{75\pi^2}. \qquad (30)$$

One can obseve from this equation that given the value of δ_H from observations, it is possible to completely constrain λ as

$$\lambda^6 \simeq \frac{75\pi^2\delta_H^2}{61^4\kappa^2\rho_0} \qquad (31)$$

We have a dimensionles number that fixes one of the parameters of the potential and can be contrasted with the

value predicted by this supergravity model when interpreted as Dark Matter [2].

If we substitute the last equation into Eq. (22), we get in principle a set of values for the constant V_0 that satisfy the field domination condition.

C. Vacuum energy-dominated regime

We consider now the regime in which the second term of Eq. (5) dominates the dynamics. In this case the slow -roll parameters Eqns. (13) are:

$$\epsilon \simeq -\frac{\lambda^2 \rho_0 e^{2\lambda\sqrt{\kappa}\phi}}{4V_0} \tag{32}$$

$$\eta \simeq \frac{\lambda^2 \rho_0 e^{\lambda\sqrt{\kappa}\phi}}{2V_0}. \tag{33}$$

We find ourselves here with the fact that ϵ is negative, that is, with a period of deflation [17]. Such behviour has already been predicted for these models before [1].

It is necessary to point out that this is a consequence of the modification of the dynamics. The definition of ϵ for the Randall–Sundrum II cosmology in the high-energy limit is not positive definite as in the standard cosmology case. Thus a stage of accelerated contraction for this regime on the potential is only a result of the modification in the field equations.

Following the evolution of the dynamics with the potential Eq. (5), from a region where ϕ dominates, to a stage in which the energy V_0 drives the behaviour of the expansion, we observe a primordial inflationary expansion that erases all information concerning any process that the ψ field might have undergone under the influence of the potential Eq. (4). The intermediate regime, in which both terms in Eq. (5) are of the same order, produces further expansion. Finally the field ϕ reaches a value on the potential that comences a stage of accelerated contraction. This value is obtained when the denominator in Eq. (16) changes sign:

$$\sqrt{\kappa}\phi_d = \frac{\ln 2}{\lambda}, \tag{34}$$

corresponding to the value of the vacuum-dominated regime. Such process takes place when the field ϕ takes values below $\sqrt{\kappa}\phi_d$. Eq. (34) is equivalent to $V(\phi_d) = 0$. Thus the balance of the terms in Eq. (5) and the sign of V_0 determine the place where deflation starts as the point where the potential crosses the ϕ axis.

In principle, there would not be a physical reason that could prevent this deflationary stage to stop. But the argument mentioned before, concerning the modification of the dynamics applies again . We observe ϵ has a dependence on the field and therefore undergoes an evolution accordingly. The condition to end deflation is $\epsilon = -1$, in oposition to inflation. In consequence, we can also find from the first of equations (32) a value of ϕ corresponding to this;

$$\epsilon = -1 \Rightarrow \quad \sqrt{\kappa}\phi_e = \frac{1}{2\lambda} \ln\left(\frac{4V_0}{\lambda^2 \rho_0}\right). \tag{35}$$

Where this time, the subscript “e”, indicates the end of deflation. This value, as we can see, depends on V_0 and λ.

The choice of V_0 parameter will be given in section IV where we give different values acording to the conditions we find in the following. We shall see whether or not an inflationary stage takes place under the value of λ found in the previous analysis.

D. Intermediate Regime

The intermediate regime corresponds to the region where both terms in Eq (5) are of the same order. In order to obtain a bound on the values of V_0 that satisfy the COBE constrain (26), we need to solve numerically Eqs. (18) and (19).Since we have that both terms in Eq. (5) are important, this means that the exponential is of $\mathcal{O}(1)$, thus we can expand it in Taylor series as $\exp(\lambda\sqrt{\kappa}\phi) \simeq 1 + \lambda\sqrt{\kappa}\phi$ and arrive to a value of ϕ_e equal to

$$\begin{aligned} \phi_e \; &= \; \frac{(4B)^{1/3}}{3\lambda V_0} + \left(2 + \frac{\lambda^2\rho_0}{3V_0}\right)\frac{\lambda\rho_0 4^{2/3}}{\kappa B^{1/3}} \\ &+ \frac{2\lambda\rho_0}{3\sqrt{\kappa}V_0} + \frac{1}{\lambda\sqrt{\kappa}} \end{aligned} \tag{36}$$

where B now is given by

$$\begin{aligned} B = \frac{\lambda^2\rho_0}{\kappa^{3/2}} \big(&18\lambda^2\rho_0 V_0 + 27V_0^2 + 2\lambda^4\rho_0^2 \\ &+ \sqrt{(3V_0)^3(4\lambda^2\rho_0 + 27V_0)}\big) \end{aligned} \tag{37}$$

In order to have a real scalar field, we find two bounds for the values that V_0 can take on following from the roots in the previous expressions:

$$V_0 < -\frac{1}{6}\lambda^2\rho_0, \qquad V_0 > -\frac{4}{27}\lambda^2\rho_0 \qquad V_0 > 0. \tag{38}$$

The first bound coincides with the value given by Eq (24) needed to have real values in condition (22). It is an upper bound for the allowed values of V_0 in expression (19). On the other hand, we have from Eqns. (19) and (20) that positive values of V_0 also satisfy that there exists a real value of ϕ_e in the intermediate regime. But V_0 cannot be 0 otherwise many of the equations we have been looking at would be undefined. So we have an interval of allowed values of V_0 as:

$$V_0 < -\frac{1}{6}\lambda^2\rho_0, \qquad V_0 > 0. \tag{39}$$

If $V_0 > 0$, Eq. (32) is negative and we have the period of deflation mentioned before. But $V_0 < -1/6\lambda^2\rho_0$ means

that even in the region of vacuum domination ϵ can be positive and we return to the usual picture of inflation. However, having chosen a value of V_0 below this bound, Eqns. (16) and (27) become negative, thus giving a period of deflation translated to the epochs of field domination and the intermediate regime.

We can then choose values for V_0 below $-1/6\lambda^2\rho_0$ being in a period of deflation for the first two regimes ending with a stage of inflation for the V_0 dominated region. Eqns. (20) and (19) still have real values because the approximation made in this section to find the upper limits of V_0, is a lower bound on Eqns. (19) and (20). This is in fact redundant since we have also said that both terms in the potential (5) are of the same order, so the expansion is valid for the general case.

Once the choice of V_0 is done, we can solve numerically to find a value for ϕ_N in Eq. (18), then it is introduced into Eq. (25) and can be accepted or rejected depending on whether or not it fullfils the left-hand side. This value depends on ρ_0, the brane tension, and we present the results for different values of it considering that in order to have no incompatibilities with nucleosynthesis the brane tension must satisfy $\rho_0 \geq 2\mathrm{MeV}^4$ [5], the authors take the number $1\mathrm{MeV}^4$, the difference arises due to the change of notation. We also check that the choice on ρ_0 satisfies the COBE constrain.

The results are shown in table II.

E. Vacuum-dominated region revisited

Following the argument in the preceeding paragraph, it would be possible to continue with the usual analysis to find the value of ϕ that finishes inflation, in the vacuum-dominated region provided V_0 is negative and calculate the number of e-foldings and the use Eq. (25) in the region of vacuum domination to find a constrain on V_0. We find from equation (32) that

$$\sqrt{\kappa}\phi_e = \frac{1}{2\lambda}\ln\left(-\frac{4V_0}{\lambda^2\rho_0}\right), \tag{40}$$

and from Eq (15) that

$$\sqrt{\kappa}\phi_{60} \simeq -\frac{1}{\lambda}\ln\left[\left(-\frac{\lambda^2\rho_0}{4V_0}\right)^{1/2} - \frac{30\lambda^2\rho_0}{2V_0}\right]. \tag{41}$$

And finally from (25):

$$(-V_0)^{3/2} - 60\lambda\rho_0^{1/2}V_0 = \frac{\sqrt{75}\pi\delta_H\rho_0}{\kappa} \tag{42}$$

This equation can be solved numerically to give a value of V_0 that is in accordance with the COBE constrain for the perturbations. As we shall see later, this process will not be applied to the case of vacuum domination, since we find ϵ to be a decreasing function of time, therefore for this case inflation never ends. Therefore it is not possible to find a value of the parameter V_0 satisfying this condition in the case of vacuum domination hence the values of V_0 coresponding to this regime are ruled out by observations. So in fact, the value of ϵ corresponding to 1 indicates in this case the place where inflation starts to take place as from there onwards we will have that $\epsilon < 1$

F. Field Equations

In this subsection we solve the system of field equations. The numeric results are presented in the next section. Integrating Eqns.(12) for the potential (5) yields:

$$a(t) = a_0 \exp\left[-\frac{V_0}{\lambda^4 t\sqrt{3\kappa\rho_0^3}}(\lambda^4\kappa\rho_0 t^2 + 24e^{-2})\right] \tag{43}$$

and

$$\sqrt{\kappa}\phi(t) = -\frac{2}{\lambda} + \frac{1}{\lambda}\ln\left(\frac{48}{\lambda^4\kappa\rho_0 t^2}\right). \tag{44}$$

One can immediately see that the behaviour of the field does not depend on the value of the parameter V_0, but only on λ whose value is given by Eq. (31). From this solution and its plot, one can check that indeed the field is a decreasing function of time, having the same bahaviour regardless of the regime it is in. The scale factor shows little dependance on the value of V_0 as shown in the plots. Following an increasing behaviour for $V_0 > 0$. Figures 3, 4, 5 show that for $V_0 > 0$, ϵ is a growing positive function which corresponds to an inflationary stage as expected from the analysis of the previous sections. The acceleration factor is also shown, and one can observe that for the range used in the plots $\ddot{a}/a$ changes sign before ϵ reaches 1 in the same interval.

We have plotted for completeness the scale and acceleration factors as well as ϵ in Fig.6 for a negative value of V_0. The scale and acceleration factors change the sign of their slope at $\lambda^2(\kappa\rho_0)^{1/2}t = 1.8$ which is the same value as that from Eq. (34) indicating the start of the vacuum-dominated regime. So one ends with a stage of inflation after deflation in the other two regimes. We find that ϵ is a positive decreasing function of time according with what was found. This means that although there is a period of inflation, after deflation, it will never end and there is no meaning in calculating the value of the potential from Eq. (42) satisfying the COBE constrain since there is no value of the field corresponding to 60 e-folds before the end of inflation. The case of vacuum domination with a negative potential is not realistic for this model with our approximations.

The bounds that the field needs to meet in order to have inflation for the intermediate regime are presented in table III. The value of the parameter V_0 in the region of vacuum domination remains unconstrained, therefore it is not possible to give a bound on the value of the field for the beginning of inflation.

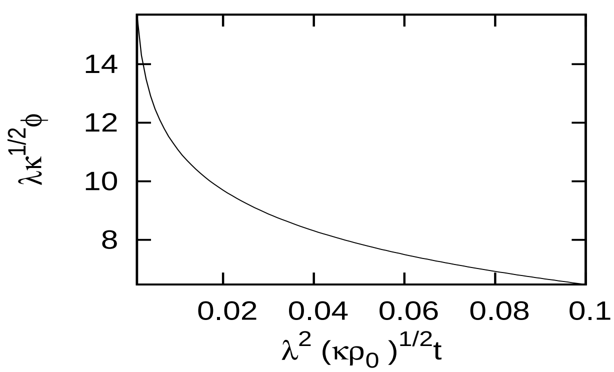

FIG. 2: The behaviour for the scalar field during inflation.

IV. RESULTS

Before starting with the numeric results for the three regions just analized, we summarize what we have found so far. They are shown schematically in table I.

Despite the fact that $V_0 > 0$ gives a positive value for Eq. (27), it does not correspond to the region of field domination. So one has to employ the value of $V_0 > 0$ in Eqns. (16) and (19) not in (28). That is, in the intermediate regime. We have checked that $V_0 > 0$ does not meet the condition (22) even for very small values of V_0 compared to unity. Instead, the smaller this parameter is, the closer to 2 is Eq. (22). So we are left with only two regions where we have inflation. The intermediate regime for $V_0 > 0$, and the region of vacuum domination for $V_0 < -1/6\lambda^2\rho_0^2$.

We solved numerically the corresponding equations in the intermediate regime and found the values shown in table II. For this, we have taken that $\kappa \simeq 25/m_{Pl}^2$ [5].

Table II shows 3 values of the potential that are in good agreement with the value of the density contrast δ_H. The numbers that appear in the third column conrrespond to $1/100\lambda^2\rho_0$, $1/120\lambda^2\rho_0$ and $1/150\lambda^2\rho_0$ respectively. Bigger values in the denominators seem to lead the decimals in the density contrast closer to 1.9×10^{-5}. We keep only these level of accuracy as a good approximation to the ideal value of the potential [15].

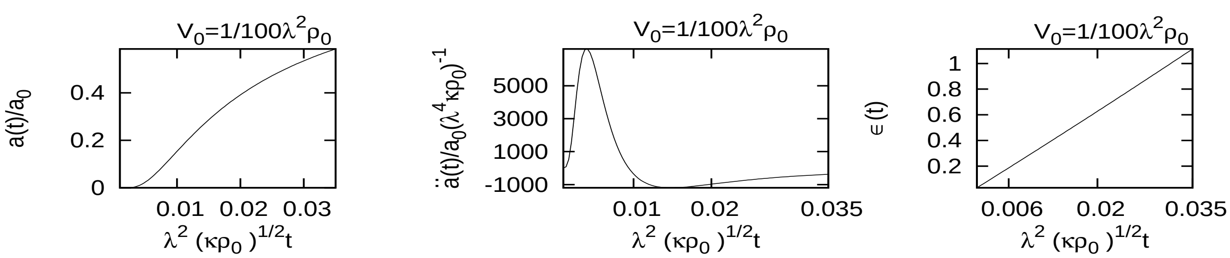

FIG. 3: On the left hand side the scale factor shows an inflationary behaviour but the acceleration factor, on the center, grows and decreases in the same interval. On the right hand side we plot the inflationary parameter ϵ.

V. CONCLUSIONS

In this work we have seen the conditions that the parameters of potential (5) have to fulfill in order to have early universe inflation, using the same scalar field potential as in [2] in order to have a unified picture between

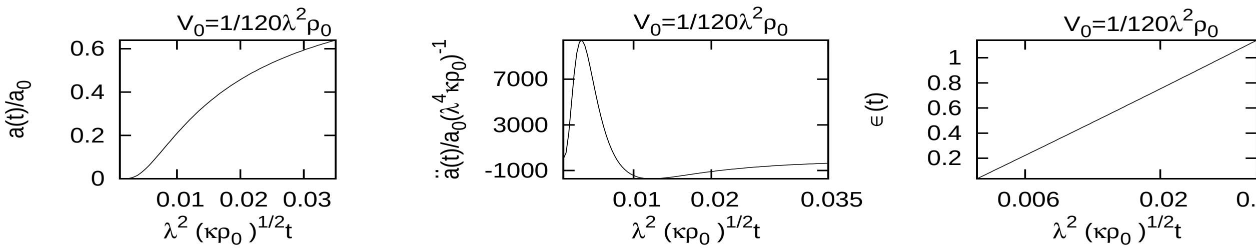

FIG. 4: In this case the scale factor (lhs) increases more rapidly than the previous case and the acceleration factor (center) reaches higher numbers within the same interval. On the rhs, again we plot the ϵ parameter.

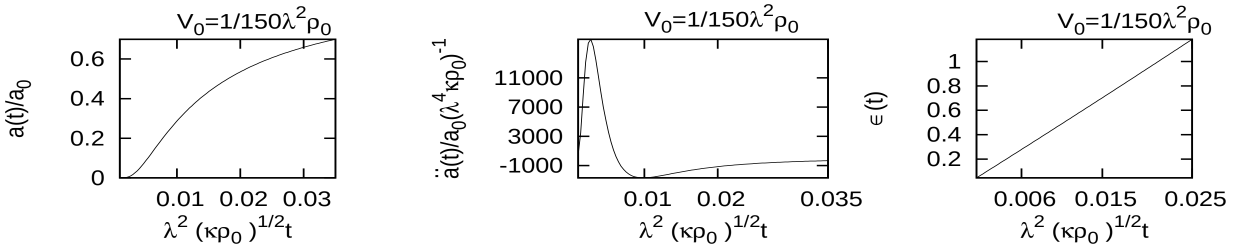

FIG. 5: Again in this case the scale factor (lhs) increases more rapidly than in the first case and the acceleration factor (center) reaches much higher numbers within the same interval. On the rhs, again we plot the ϵ parameter.

TABLE I: Results for the sign of the first slow-roll parameter ϵ according to the choice of V_0 for the three different regions of potential (5).

Region	$V_0 > 0$	dynamics	$V_0 < -\frac{1}{6}\lambda^2\rho_0$	dynamics
ϕ-dominated	$\epsilon > 0$, ϕ_e real	inflation	$\epsilon < 0$	deflation
Intermediate	$\epsilon > 0$, ϕ_e real	inflation	$\epsilon < 0$	deflation
Vacuum-dominated	$\epsilon < 0$	deflation	$\epsilon > 0$, ϕ_e real	inflation

TABLE II: Results for V_0 in the intermediate regime, for three different values of ρ_0.

$\rho_0 \times 10^6$, (eV^4)	$\lambda \times 10^{14}$	$V_0 \times 10^{34}$, (eV^4)	$\phi_e \times 10^{13}$, (eV)	$\phi_{60} \times 10^{13}$, (eV)	δ_H, $\times 10^{-5}$
2	8.2	1.4	1.5	2.7	1.9
		1.1	1.6	2.8	1.9
		0.9	1.6	2.8	1.9
4	7.4	2.2	1.7	3.1	1.9
		1.8	1.8	3.1	1.9
		1.4	1.9	3.2	1.9
6	6.9	2.8	1.9	3.3	1.9
		2.4	1.9	3.3	1.9
		1.9	1.9	3.4	1.9

TABLE III: Bounds for the scalar field ϕ multiplied by the Planck mass, for V_0 positive in eV units.

$\rho_0 \times 10^6$, (eV^4)	$\lambda \times 10^{14}$	V_0, (eV^4)	$\phi > \times 10^{-15} \times m_{Pl}$, (eV)
2	8.2	$\frac{1}{100}\lambda^2\rho_0$	1.3
		$\frac{1}{120}\lambda^2\rho_0$	1.3
		$\frac{1}{150}\lambda^2\rho_0$	1.4
4	7.4	$\frac{1}{100}\lambda^2\rho_0$	1.4
		$\frac{1}{120}\lambda^2\rho_0$	1.5
		$\frac{1}{150}\lambda^2\rho_0$	1.5
6	6.9	$\frac{1}{100}\lambda^2\rho_0$	1.5
		$\frac{1}{120}\lambda^2\rho_0$	1.6
		$\frac{1}{150}\lambda^2\rho_0$	1.7

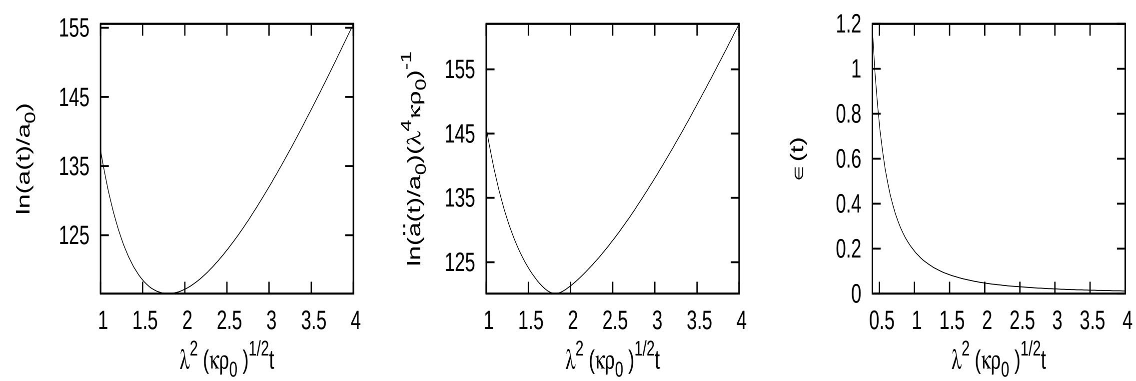

FIG. 6: The plot of the scale factor, the acceleration factor and ϵ for V_0 negative, here we use $V_0 = -56\lambda^2\rho_0$

inflation and structure formation. We find that the value of the parameter λ can be fixed in the region of field domination and that there can be two possibilities for the sign of the parameter V_0. Each of them determine different dynamics in the evolution of the field equations. A positive sign leads to a period of inflation followed by one of deflation, whereas the opposite sign implies the contrary. In the first case, the values of V_0 we have found as viable to meet the COBE constrain, are not in agreement with those found in [2] by several orders of magnitude. This behaviour seems to be generic in superstrings theory, implying that if we would like to relate the moduli fields with the inflaton, dark energy or dark matter, the model could fit observations either during the inflationary epoch or during the structure formation, but the challenge is to derive a model which fit our observing universe during the whole history of the universe. Otherwise, superstring theory have to give alternative candidates for these fields and explain why we do not see the moduli fields in our observations. Two important points to notice are that the analysis employed in this work has been made with the assumption that the field ψ or axion does not play a significant role in the dynamics and the slow-roll approximation has simplified the equations further. This behaviour is observed in the analysis carry out in [2], where it is shown that the axion field could remain as a subdominant field. Nevertheless, even when this behaviour remains so until redshifts beyond 10^6, the axion could have a different behaviour beyond this redshifts. This is indeed a very strong assumption which is not well justified completely, but our work is a first approximation to solve the problem and we are aware that a more general analysis including the axion field needs to be done.

The second case corresponding to vacuum domination has proved unrealistic to have a viable model of inflation since this process does not end and we do not have other mechanism to finish it as in the usual hybrid inflation scenario. It would follow from this that a more general analysis is needed in order to determine whether the consequences of our assumptions are important or not.

VI. ACKNOWLEDMENTS

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